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Transient modelling of a linear induction launcher-type coil gun with two-dimensional cylindrical finite-difference time domain method

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Abstract: Two-dimensional (2D) simulation of a linear induction launcher-type coil gun is performed by the cylindrical finite-difference time domain (FDTD) method. Maxwell's equations in cylindrical coordinates are used to formulate the problem. Circular symmetry is assumed for reducing the 3D problem to a 2D one. A singularity that appeared at $r = 0$ is extracted by using the integral formulation of the FDTD method. A special quasi-static approximation of the FDTD method is applied in the cylindrical problem space which is terminated with the second-order Mur absorbing boundary condition. The armature movement in the FDTD method is treated by the quasi-steady-state method. An experimental model was built, and the muzzle velocity was measured for comparison with the calculated value. Good agreement was obtained.

1 Introduction

Electromagnetic launchers are of interest today because they open wide a large number of possibilities for use in their areas of application [1]. There are many types, such as rail guns, coil guns and hybrid versions, but the linear induction launcher (LIL)-type coil guns stands out as one of the more important ones because of its advantages over the others [2, 3]. In this sense, the mathematical modelling of the LIL type coil gun has become an increasingly important topic. It is needed to support the possibilities of the LIL for design and improvement features. Because the LIL-type coil gun works in a transient regime, the time domain modelling of it is a more suitable way for the analysis. If one looks through the literature about the mathematical modelling of the LIL-type coil gun, the analytical and numerical works are found presenting solutions of the problem [4]. The analytical solutions are based on two fundamental approaches: the transmission line method [5–8], and the current sheet method [9]. The hybridisation of these two methods is also proposed [10]. Although the finite-element method [11, 12] and its

combination with the boundary element method [13] have been used as numerical approaches to solve LIL-type coil gun problems, time-domain numerical methods such as the finite-difference time domain (FDTD) method have not been used to solve the coil gun problems as yet [14].

In this paper, for the first time, the cylindrical FDTD method is applied to the coil gun problem. This is a suitable approach because the FDTD method is capable of modelling the transient behaviour of launching phenomena. The quasi-static (QS) approximation in cylindrical coordinates is considered and applied in the FDTD method for the solution because of the low frequencies of the power sources. Armature movement in the FDTD method is treated by the quasi-steady-state method [15].

This paper is organised as follows. In Section 2, the details of the electromagnetic modelling of the LIL-type coil gun in cylindrical coordinates with the QS approximation are given. The application of the FDTD method to the coil gun problem is shown and the FDTD update equations are extracted. Critical issues such as the singularity extraction at

$r = 0$, the QS approximation and the absorbing boundary condition are discussed for the FDTD application. In Section 3, a set of numerical results (the field and force distribution, velocity profiles according to time) are shown. In Section 4, the details are given of an experimental model that was built and used to measure muzzle velocity for comparison with calculated values. The last section gives conclusions and directions for future work.

2 Formulation of the problem and FDTD update equations

The formulation of the coil gun problem is started with the assumption of circular symmetry ($\partial/\partial\phi = 0$) in cylindrical coordinates. It is assumed that the source current density J_ϕ^{source} is only directed azimuthally in order to model the current flowing in the driving coil windings. In this case, Maxwell's equations can be evaluated as a separate set

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = \varepsilon \frac{\partial E_\phi}{\partial t} + \sigma E_\phi + J_\phi^{\text{source}} \quad (1)$$

$$\frac{\partial E_\phi}{\partial z} = \mu_0 \frac{\partial H_r}{\partial t} \quad (2)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) = -\mu_0 \frac{\partial H_z}{\partial t} \quad (3)$$

where r , ϕ and z denote radial, angular and longitudinal cylindrical coordinates, respectively, and t is the time variable. E_ϕ is the electric field component in the angular direction, H_r and H_z are the magnetic field components in the radial and longitudinal directions, respectively. ε (F/m) and σ (S/m) are dielectric permittivity and conductivity in the problem space, μ_0 is the free-space magnetic permeability. The medium, except for the projectile and the driving coils, is assumed to be free space. Using the equations given above, one can extract the cylindrical FDTD update equations as

$$H_r|_{i,k+1/2}^{n+1/2} = H_r|_{i,k+1/2}^{n-1/2} + \frac{\Delta t}{\mu \Delta z} (E_\phi|_{i,k+1}^n - E_\phi|_{i,k}^n) \quad (4)$$

$$H_z|_{i+1/2,k}^{n+1/2} = H_z|_{i+1/2,k}^{n-1/2} - \frac{\Delta t}{\mu r_{i+1/2} \Delta r} (r_{i+1} E_\phi|_{i+1,k}^n - r_i E_\phi|_{i,k}^n) \quad (5)$$

$$E_\phi|_{i,k}^{n+1} = \frac{\gamma_1}{\gamma_2} E_\phi|_{i,k}^n + \frac{\gamma_3}{\gamma_2} \left(\frac{H_r|_{i,k+1/2}^{n+1/2} - H_r|_{i,k-1/2}^{n+1/2}}{\Delta z} \right) - \frac{\gamma_3}{\gamma_2} \left(\frac{H_z|_{i+1/2,k}^{n+1/2} - H_z|_{i-1/2,k}^{n+1/2}}{\Delta r} \right) - \frac{\gamma_3}{\gamma_2} J_\phi|_{i,k}^{n+1/2} \quad (6)$$

where $\gamma_1 = 1 - (\sigma_{ik} \Delta t / 2 \varepsilon_{ik})$, $\gamma_2 = 1 + (\sigma_{ik} \Delta t / 2 \varepsilon_{ik})$ and $\gamma_3 = \Delta t / \varepsilon_{ik}$. The armature movement in the FDTD method is treated by the quasi-steady-state approach [15].

2.1 Treatment of singularity at $r = 0$

A singularity that appeared at $r = 0$ is extracted by using the integral form of Maxwell's equations, and then the special FDTD update equation at $r = 0$

$$H_z|_{0,k}^{n+1/2} = H_z|_{0,k}^{n-1/2} - \frac{4\Delta t}{\mu_0 \Delta r} E_\phi|_{1,k}^n \quad (7)$$

is applied [16].

2.2 Quasi-static FDTD(QS-FDTD) method

The classical FDTD application for the modelling of the LIL-type coil gun problem is not simply straightforward because very high power voltage or current sources in the coil gun applications operate generally at low-frequency regime, today. It means that the wavelength is large compared to the real problem space dimensions. In the FDTD method, this gives trouble for the discretisation of the problem space. In order to obtain acceptably accurate results, the discretization has to be fine enough in the sense of the electrical length of the problem space. This necessity forces the number of FDTD cells to be high, resulting in very small time steps, which causes the computation time to be very long. In this case, it may not be possible to solve the coil gun problem with the limitation of the classical FDTD method by using the present personal computers. This difficulty is overcome with the QS-FDTD method which satisfies the QS condition (approximation) in the simulations. The QS condition (the displacement current is smaller than the conduction current) can be formulated as

$$\frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) \ll \sigma \mathbf{E}(\mathbf{r}, t) \Rightarrow i\omega \varepsilon \mathbf{E}(\mathbf{r}) \ll \sigma \mathbf{E}(\mathbf{r}) \Rightarrow |\omega \varepsilon| \ll |\sigma| \quad (8)$$

Whenever the above condition is satisfied, the velocity of wave propagation can be decreased resulting in bigger time steps. Thus, the computation time for the LIL-type coil gun QS-FDTD simulation will be in an acceptable range for present personal computers.

2.3 Stability for cylindrical QS-FDTD method

The QS-FDTD method will be stable for all the time progress whenever the stability condition for the two-dimensional (2D) cylindrical QS-FDTD, given below [17], is satisfied

$$\Delta t \leq \frac{\sqrt{\alpha}}{c \sqrt{(1/\Delta r)^2 + (1/\Delta z)^2}} \quad (9)$$

where Δt is the time increment, α is the QS parameter, c is the velocity of light, Δr and Δz are the space increments along the radial and the longitudinal direction, respectively.

2.4 Application of absorbing boundary condition

The LIL-type coil gun modelling with the FDTD method is an open-space problem which means that the absorbing boundary condition has to be used along the border of the solution space in order to terminate the full problem space. Because the FDTD update equations are used in cylindrical coordinates, the absorbing boundary condition has also to be extracted in cylindrical coordinates. The second-order Mur type absorbing boundary condition is used in this work as an absorbing boundary condition along the surfaces of the $z = 0$, $z = z_{\max}$ and $r = r_{\max}$ in cylindrical coordinates because the $r = 0$ plane is not an open boundary [18].

3 Numerical results

A schematic view of the LIL-type coil gun is shown in Fig. 1. Because of the axial symmetry of the armature, only half of the LIL-type coil gun is used in the model. This decreases the computation time and memory required in the QS-FDTD solution. The design parameters of the LIL-type coil gun considered in the simulations are given in Table 1.

The source frequency is applied as 250 Hz, the maximum producible frequency in our experimental set-up. The current of the drive coils is assumed to flow only in the azimuthal

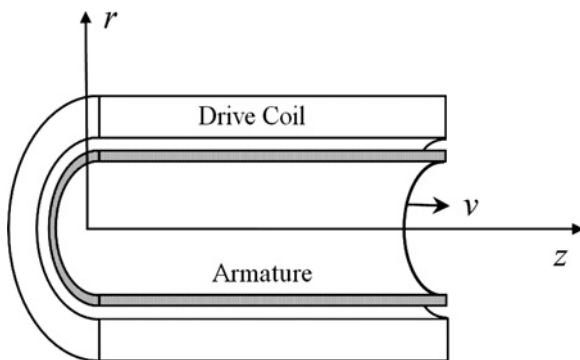


Figure 1 Schematic view of the LIL-type coil gun

Table 1 Design parameters of the coil gun

| | |
|--|------------------------|
| length of the drive coils | 0.2 m |
| length of the armature | 0.2 m |
| inner radius of the armature | 0.014 m |
| outer radius of the armature | 0.015 m |
| inner radius of the drive coil | 0.016 m |
| outer radius of the drive coil | 0.031 m |
| distance between the drive coil and the armature | 0.001 m |
| number of turns for the drive coil | 100 |
| pole-pitch | 0.1 m |
| conductivity of the armature (aluminium) | 3.96×10^7 S/m |
| mass of the armature | 0.1 kg |
| current, RMS | 65 A |

direction in the form of a travelling wave

$$I = I_m \cos(kz - \omega t) \quad (10)$$

where I_m is the peak value of the current, k is the wave number and $\omega = 2\pi f$ is the angular frequency. The relation between the wave number and the pole pitch is

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{\tau} \Rightarrow \tau = \frac{\lambda}{2} \quad (11)$$

In the QS-FDTD analysis, the QS parameter is chosen as $\alpha = 5 \times 10^{11}$ in order for good modelling of magnetic diffusion through conductive armature [17]. The unit cell length along the radial and the longitudinal directions are $\Delta r = 0.001$ m and $\Delta z = 0.001$ m, respectively. In this case, the number of cells along the radial and longitudinal directions become $(N_r, N_z) = (40, 425)$. The unit time step Δt with the consideration of the QS-FDTD stability condition is 1.5×10^{-6} s. The total number of the

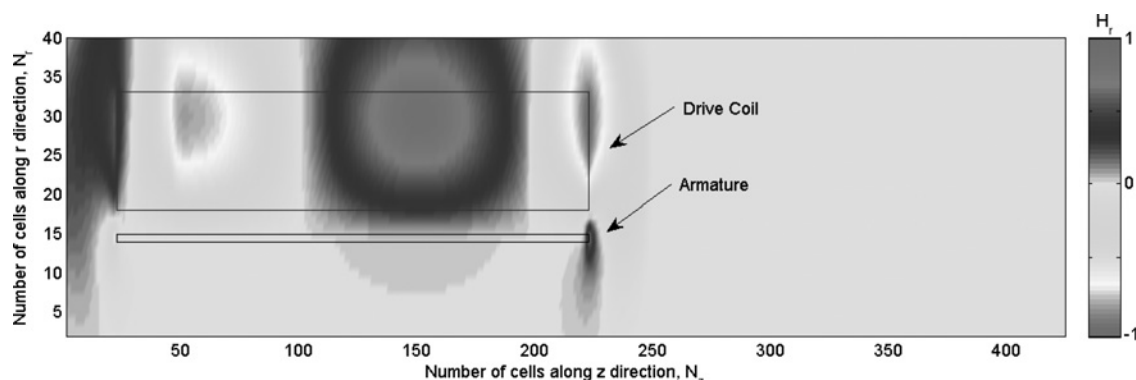


Figure 2 Normalised field distribution of the radial magnetic component H_r at the beginning of the QS-FDTD solution

QS-FDTD time iterations corresponding to the desired number of periods is 3500. With the above-mentioned parameters, the calculated normalised field distribution of the radial magnetic component H_r at the beginning of the QS-FDTD simulation is shown in Fig. 2 where the excited travelling waves with the different amplitude level of magnetic field on the armature are shown with different colours, respectively. These travelling waves built up the Lorentz force on the conductive armature. In Fig. 3, the

normalised field distribution of the radial magnetic component H_r at the end of the QS-FDTD simulation (the projectile is fully launched) is shown. Because the simulation is in the time domain, all the results can be evaluated at the desired time or can be observed along the full simulation time. The assumption of the azimuthal projectile current density direction results in the presence of the r and z components of the magnetic field, only. In this case, the induced projectile current causes the Lorentz force

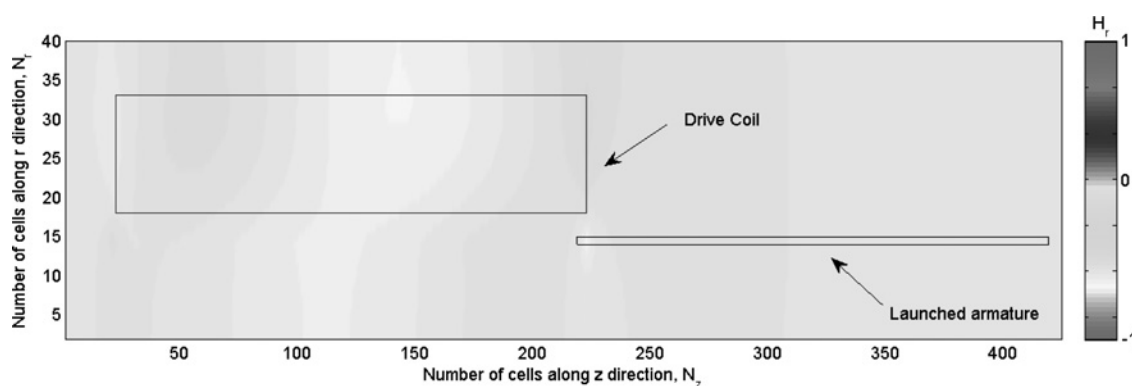


Figure 3 Normalised field distribution of the radial magnetic component H_r at the end of the QS-FDTD solution

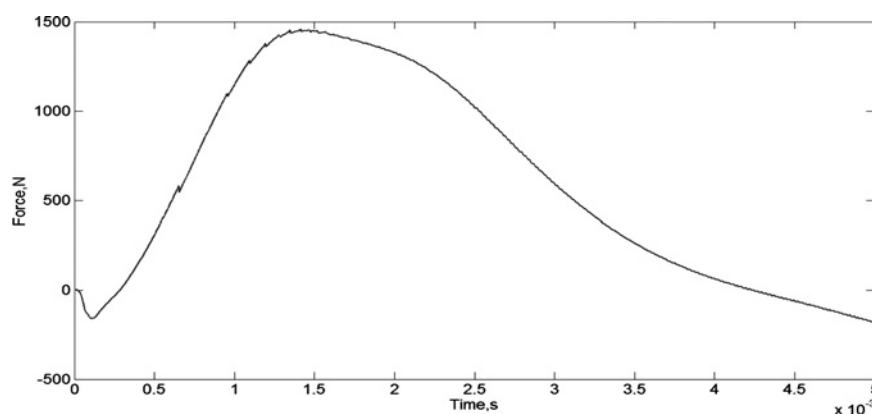


Figure 4 Calculated time dependency of the total force acting on the projectile

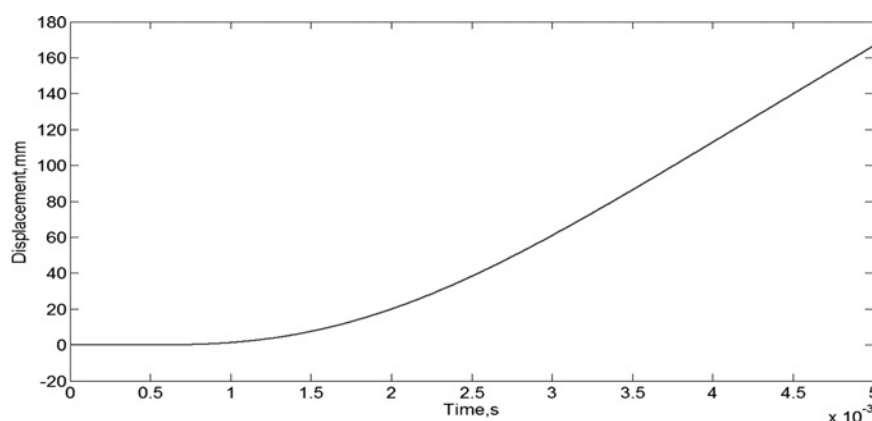


Figure 5 Calculated displacement according to time

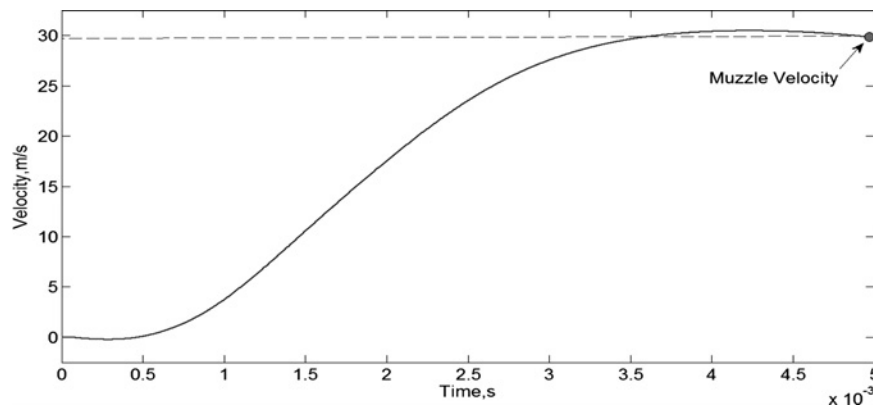


Figure 6 Velocity dependency according to time

acting on the armature to be

$$\begin{aligned} F(r, t) &= J_{\phi}(r, t) \times \mu_0 H(r, t) \\ &= \mu_0 J_{\phi} H_z r - \mu_0 J_{\phi} H_r z \end{aligned} \quad (12)$$

where it is clear that only the r and z components of the force are present. Whenever the first one will try to squeeze the projectile along the radial direction, the second one will be responsible for the launching along the axial direction. The calculated total force dependence on time acting on the projectile is given in Fig. 4. According to Fig. 4, negative values are observed at the beginning. This is due to phase mismatch of the current in the drive coils and armature until the travelling wave is built up, completely. Using the relation between the calculated force and the projectile mass as below

$$F_z = ma \Rightarrow F_z = m \frac{d^2}{dt^2} Z(t) \quad (13)$$

the calculated displacement dependence $Z(t)$ on time is shown in Fig. 5. The velocity dependence on time is also shown in Fig. 6.

4 Experimental set-up for validation

The considered LIL-type coil gun is an air-cored electromagnetic launcher. The operating concept is based on the principle of the classical induction machine. The experimental model of the LIL-type coil gun is shown in Fig. 7. The system consists of three main parts: one is the launcher itself, the second is the power supply and the third is the projectile. The launcher has six coils and fed by a three-phase generator. Polyphase excitation of the coils constituting the barrel is designed to create an electromagnetic wave packet which travels from the breach to the muzzle as similar to the field distribution of magnetic field in Fig. 2. The projectile is a hollow-conducting cylinder (sleeve). The input parameters used in

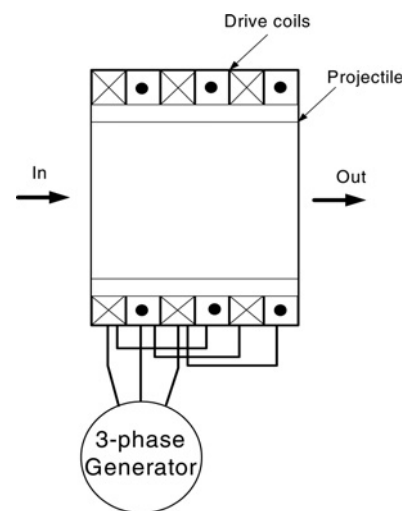


Figure 7 Experimental model for the LIL-type coil gun

the experiment are the same as in Table 1 with the applied voltage of 1500 V.

Using the system explained, the drive coils are energised by the generator. Fig. 8 shows the launcher set-up with the velocity sensor, which is located at the end of the muzzle.

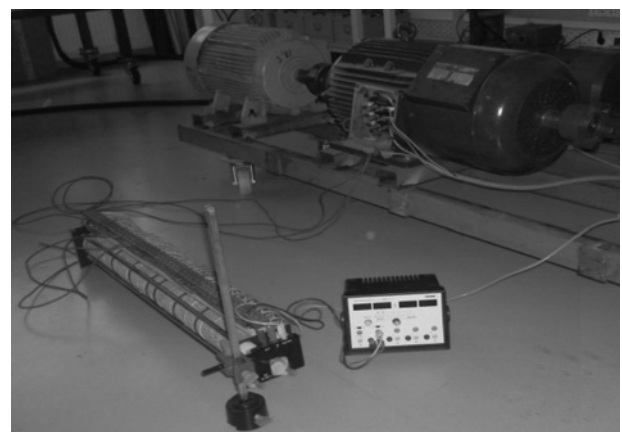


Figure 8 Whole launcher set-up with the velocity sensor

This sensor is used to measure instant linear velocity and the measurement error of the sensor is equal to 0.1 %. It is an optical linear sensor and counts the time while the projectile is exiting from the muzzle until it leaves the muzzle completely. When the drive coils are energised, the measured muzzle velocity is 33.17 m/s. According to Fig. 6, the calculated muzzle velocity is 29.78 m/s. This is reasonably in good agreement with the experimental one.

5 Conclusion

The LIL-type coil gun problem is solved by using the FDTD method in 2D cylindrical coordinates. The problem is formulated in the $r-z$ -plane with the assumption of axial symmetry. A singularity that appeared at $r=0$ is removed by using the integral form of Maxwell's equations. The second-order Mur type absorbing boundary condition is used in order to terminate the FDTD problem space. Specially, the QS approximation is considered in the FDTD method because of the low-frequency sources causing the long computation time in the classical FDTD method. The armature movement in the FDTD method is treated by the QS state approach. The force that acts on the projectile because of the induced current is calculated as a function of time. Then, the displacement and the velocity dependence on time are shown. The experimental work is also performed using the same input parameters as in the QS-FDTD simulation. Experimental and calculated results for the muzzle velocity were in good agreement. As a future work, first of all, 3D QS-FDTD solution of the coil gun problem is aimed to solve for removing the limitation of the axial symmetry assumption.

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