

Laplace Transform Pairs

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt \quad , \quad s \in C$$

Number	$f(t)$	$F(s)$	Poles
1	$\delta(t)$	1	none
2	1	$\frac{1}{s}$	simple
3	e^{-at}	$\frac{1}{s+a}$	simple
4	te^{-at}	$\frac{1}{(s+a)^2}$	multiple
5	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	multiple
6	$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$	imaginary
7	$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$	imaginary
8	$e^{-at} \cos(\omega t)$	$\frac{s+a}{(s+a)^2 + \omega^2}$	complex
9	$e^{-at} \sin(\omega t)$	$\frac{\omega}{(s+a)^2 + \omega^2}$	complex

Laplace Transform Properties

Property	Name
$L\{af(t) + bg(t)\} = aF(s) + bG(s)$	linearity
$L\{e^{-at}f(t)\} = F(s + a)$	frequency shift
$L\{f(t - T)u_s(t - T)\} = e^{-sT}F(s)$	time shift
$L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$	scaling
$L\{\dot{f}(t)\} = sF(s) - f(0^-)$	first derivative
$L\{\ddot{f}(t)\} = s^2F(s) - sf(0^-) - \dot{f}(0^-)$	second derivative
$L\{f^{(n)}(t)\} = s^nF(s) - \sum_{k=1}^n s^{n-k}f^{(k-1)}(0^-)$	general derivative
$L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$	integral
$f(0^+) = \lim_{s \rightarrow \infty} \{sF(s)\}$	initial value
$f(\infty) = \lim_{s \rightarrow 0} \{sF(s)\}$	final value, $sF(s)$ stable